Disappearances of uncoupled modes in two-dimensional photonic crystals due to anisotropies of liquid crystals

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We demonstrate disappearances of uncoupled modes in two-dimensional photonic crystals due to anisotropies of liquid crystals theoretically. Mirror symmetry disappears in wave vectors by rotating directors of liquid crystals, which results in disappearances of uncoupled modes that cannot be excited by external plane waves. This property may provide large tunabilities in two-dimensional photonic crystals utilizing liquid crystals.

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I. INTRODUCTION

Recently, dielectric periodic structures of optical wavelength order have attracted much attention as photonic crystals from both fundamental and practical viewpoints, because novel concepts such as photonic band gaps have been predicted, and various new applications of the photonic crystals have been proposed [1–3]. In earlier work, two fundamentally new optical principles, that is, the localization of light [4–6] and the controllable inhibition of spontaneous emission of light [7–10], were considered to be the most important. The existence of photonic band gaps can be investigated by studying transmission spectra. In theoretical and experimental studies, however, it has been known that the transmission spectra are not observed in uncoupled modes, nevertheless, photonic band states exist in photonic band structures [11,12]. Plane waves and uncoupled modes are symmetric and antisymmetric under the mirror reflection, respectively. Therefore, the plane waves cannot excite the uncoupled modes.

In conventional photonic crystals composed of isotropic materials, high rotational and mirror symmetry exists in wave-vector spaces. In photonic crystals composed of anisotropic materials, however, no such symmetry generally exists. For example, there exist liquid crystals (LCs) with anisotropies whose properties can easily be changed by temperature and electric field. For many applications, it is useful to obtain tunabilities of photonic band structures through electro-optic effects. Therefore, tunable photonic crystals infiltrated with liquid crystals have been proposed [13–18].

In this paper, we demonstrate disappearances of uncoupled modes in two-dimensional photonic crystals due to anisotropies of liquid crystals. The two-dimensional photonic crystals are supposed to be composed of liquid-crystal rods with square lattices. In conventional two-dimensional photonic crystals, two modes, that is, the transversal electric (TE) mode and the transversal magnetic (TM) mode, exist. In photonic crystals composed of liquid crystals, generally, none of these two classifications of modes exists due to anisotropies of liquid crystals. Even in such photonic crystals, however, we can classify the TE and TM modes in the cases of directors of liquid crystals parallel and perpendicular to two-dimensional planes.

Therefore, we treat the case of directors of liquid crystals parallel to two-dimensional planes and the TE mode, because electric fields exist only in two-dimensional planes in the case of the TE mode, and are strongly affected by rotating directors of liquid crystals.

II. THEORY

Following discussion of Busch and John [13], we start with the wave equation satisfied by the magnetic field for two-dimensional periodic structures in order to determine photonic band structures of two-dimensional photonic crystals utilizing liquid crystals,

$$\nabla\times\{\epsilon^{-1}(\mathbf{r})\nabla\times\mathbf{H}(\mathbf{r})\}=\frac{\omega^2}{c^2}\mathbf{H}(\mathbf{r}), \tag{1}$$

where $\nabla\cdot\mathbf{H}(\mathbf{r})=0$. The dielectric tensor $\epsilon(\mathbf{r})=\epsilon(\mathbf{r}+\mathbf{R})$ is periodic with respect to the lattice vector $\mathbf{R}$ generated by the primitive translation and it may be expanded in a Fourier series on $\mathbf{G}$, the reciprocal lattice vector:

$$\epsilon_{ij}(\mathbf{r})=\sum_{\mathbf{G}}\epsilon_{ij}(\mathbf{G})\exp(i\mathbf{G}\cdot\mathbf{r})\quad(i,j=x,y). \tag{2}$$

A liquid crystal possesses two kinds of dielectric indices, that is, an ordinary dielectric index ϵ^0 and an extraordinary dielectric index ϵ^e. Light with electric field perpendicular and parallel to directors of liquid crystals has ordinary and extraordinary dielectric indices, respectively. In the case of directors of liquid crystals parallel to two-dimensional planes, the components of the dielectric tensor are represented as follows:

$$\epsilon_{xx}(\mathbf{r})=\epsilon^o(\mathbf{r})\sin^2\phi+\epsilon^e(\mathbf{r})\cos^2\phi, \tag{3a}$$

$$\epsilon_{yy}(\mathbf{r})=\epsilon^o(\mathbf{r})\cos^2\phi+\epsilon^e(\mathbf{r})\sin^2\phi, \tag{3b}$$

$$\epsilon_{xy}(\mathbf{r})=\epsilon_{yx}(\mathbf{r})=[\epsilon^e(\mathbf{r})-\epsilon^o(\mathbf{r})]\cos\phi\sin\phi, \tag{3c}$$

where ϕ is a rotation angle of the director of the liquid crystal, and the director of the liquid crystal is $\mathbf{n}=(\cos\phi,\sin\phi)$. In the isotropic case, $\epsilon^o(\mathbf{r})$ is equal to $\epsilon^e(\mathbf{r})$.

Equation (1) comprises a set of three coupled differential equations with periodic coefficients. In two-dimensional photonic crystals, we can define $\mathbf{e_G}$ as the direction which is perpendicular to the two-dimensional plane. Using Bloch's theorem, we may expand the magnetic field as

$$\mathbf{H}(\mathbf{r})=\sum_{\mathbf{G}} h(\mathbf{G})\mathbf{e_G}\exp\{i(\mathbf{k}+\mathbf{G})\cdot\mathbf{r}\} \tag{4}$$

in the case of the TE mode. Inserting Eqs. (2) and (4) into Eq. (1) and multiplying by $\mathbf{e_G}$ result in the following infinite matrix eigenvalue problem:

$$\sum_{\mathbf{G}'} H_{\mathbf{G},\mathbf{G}'}h(\mathbf{G}')=\frac{\omega^2}{c^2}h(\mathbf{G}), \tag{5a}$$

where

$$H_{\mathbf{G},\mathbf{G}'}=\epsilon_{yy}^{-1}(\mathbf{G}-\mathbf{G}')(k_x+G_x)(k_x+G_x')+\epsilon_{xx}^{-1}(\mathbf{G}-\mathbf{G}')(k_y+G_y)(k_y+G_y')-\epsilon_{xy}^{-1}(\mathbf{G}-\mathbf{G}')(k_y+G_y)(k_x+G_x')-\epsilon_{yx}^{-1}(\mathbf{G}-\mathbf{G}')(k_x+G_x)(k_y+G_y'). \tag{5b}$$

For numerical purposes, Eq. (5a) is truncated by retaining only a finite number of reciprocal lattice vectors. The main numerical problem in obtaining the eigenvalue is the evaluation of the Fourier coefficients of the inverse dielectric tensors in Eq. (5b). The best method is to calculate the matrix of Fourier coefficients of real space tensors and then take its inverse in order to obtain the required Fourier coefficients. This method was shown by Ho, Chan, and Soukoulis (HCS) [19]. The eigenvalues computed with the HCS method for 289 plane waves are estimated to be in error less than 1%.

Following Sakoda's discussion [12], moreover, we calculate transmission spectra. Magnetic field H_z in two-dimensional photonic crystals utilizing liquid crystals is satisfied in the following differential equation:

$$\frac{\partial}{\partial x}\left[\epsilon_{yy}^{-1}(x,y)\frac{\partial H_z}{\partial x}\right]+\frac{\partial}{\partial y}\left[\epsilon_{xx}^{-1}(x,y)\frac{\partial H_z}{\partial y}\right]-\frac{\partial}{\partial y}\left[\epsilon_{xy}^{-1}(x,y)\frac{\partial H_z}{\partial x}\right]-\frac{\partial}{\partial x}\left[\epsilon_{yx}^{-1}(x,y)\frac{\partial H_z}{\partial y}\right]+\frac{\omega^2}{c^2}H_z=0. \tag{6}$$

We assume that the two-dimensional photonic crystal considered here possesses 16 layers.

III. NUMERICAL CALCULATION AND DISCUSSION

Let us consider that plane waves are incident on photonic crystals, as shown in Fig. 1. We assume that photonic crystals are composed of liquid-crystal rods with square lattices, and that the background is an air region. Such a condition could be realized by silica aerogels. Silica aerogels are porous structures and diameters of pores are about 20 nm. Refractive indices of silica aerogels are about 1.03, that is, they are almost the same as that of the air. Optical absorption of the hydrophobic silica aerogels occurs around the infrared region [20]. Therefore, the hydrophobic silica aerogels are useful in the visible range. Indeed, the hydrophobic silica aerogels are used as very low refractive-index materials [21]. That is, the two-dimensional photonic crystals composed of liquid crystals can be prepared by making a periodic array of holes in the silica aerogel plate and then infiltrating liquid crystals into the holes. When diameters of the holes are about 1 μm and are much larger than those of the pores, wet liquid crystals could be trapped into the drilled holes. In making the holes in the silica aerogel plate by a laser, moreover, pores around the holes may be broken by the heat of the laser. Then, we do not have to consider the leak of liquid crystals into the pores around the holes. An experimental fabrication of tunable two-dimensional photonic crystals infiltrated with liquid crystals has already been reported [15]. By the use of silica aerogels, moreover, photonic crystals composed of air rods in liquid-crystal plates could be realized.

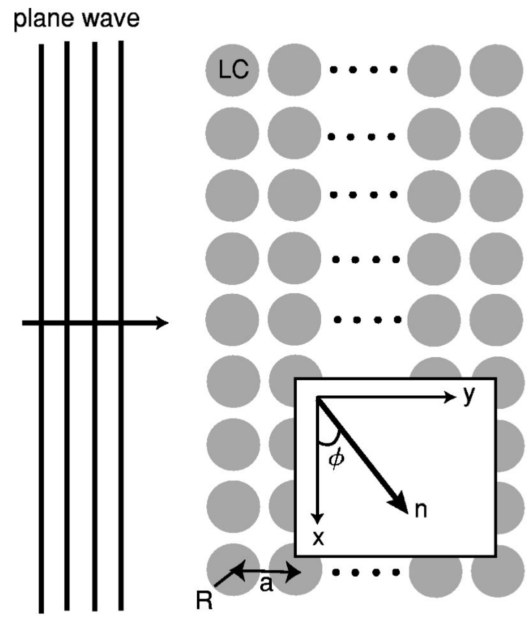

FIG. 1. Schematic model that plane waves are incident on photonic crystals composed of liquid crystals. The **n** and ϕ in the diagram indicate the director and rotation angle of liquid crystals.

When we choose high refractive-index materials as the background, we cannot obtain high anisotropies caused by liquid crystals although we can obtain high dispersion relations of frequencies and wave vectors. When we choose materials whose refractive indices are near to those of liquid crystals as the background, on the other hand, we cannot obtain high dispersion relations although we can obtain high anisotropies caused by liquid crystals. Therefore, we consider that the model we propose here is appropriate with respect to both high dispersion relations and high anisotropies.

We consider that ordinary and extraordinary refractive indices of liquid crystals are $n_{LC}^o=1.522$ and $n_{LC}^e=1.706$

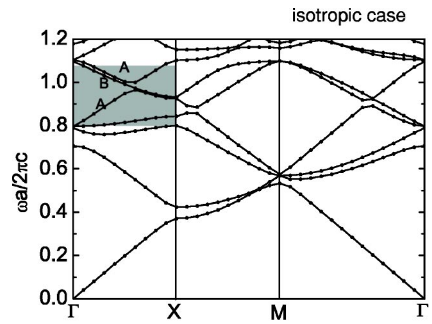

FIG. 2. Band structure of two-dimensional photonic crystals with square lattices when directors of liquid crystals are orientated at random. The average refractive index of liquid crystals is $n_{LC}^{av} = 1.583$. The A and B modes in a shaded region indicate symmetric and antisymmetric modes, respectively.

(5CB), respectively, and that a radius of a rod is $R/a = 0.4$, where a is a lattice constant. In Fig. 1, ϕ in the diagram indicates the rotation angle of directors of liquid crystals.

When an electric field is not applied, directors of liquid crystals are orientated at random. That is, anisotropies of liquid crystals disappear and liquid crystals become isotropic, and then the average refractive index of liquid crystals is $n_{LC}^{av} = (n_{LC}^{e} + 2n_{LC}^{o})/3 = 1.583$.

In Fig. 2, we display the band structure of two-dimensional photonic crystals when electric field is not applied, that is, liquid crystals become isotropic. The Γ, X, and M points indicate the high symmetric points in the first Brillouin zone.

The Γ and M, points possess C_{4v} symmetry and the X point possesses C_{2v} symmetry, respectively, in the group theory, and the ΓX, ΓM, and MX segments possess mirror symmetry, which causes symmetric and antisymmetric modes on these segments. A more detailed discussion of the symmetry in two-dimensional photonic crystals is given in Ref. [22]. For example, the A and B modes in a shaded region in Fig. 2 mean the symmetric and antisymmetric modes on the ΓX segment. The sixth band is the B mode. Symmetric plane waves on the ΓX segment cannot excite the antisymmetric B mode, and therefore, transmission spectra are not observed at the B mode. That is, the B mode is an uncoupled mode.

When an electric field is applied in two-dimensional planes, liquid crystals become anisotropic. Then, we cannot use the photonic band structures in Fig. 2 because anisotropies of liquid crystals break high symmetry. For example, profiles of the sixth band at $\phi = 0°$ and 30° are shown in Figs. 3(a) and 3(b), respectively. In the isotropic case, transmission spectra in Fig. 1 reflect on the band structure on the ΓX segment drawn by the arrow in these figures. In Fig. 3(a), it should be noted that mirror symmetry exists on the ΓX segment drawn by the arrow. Therefore, we can classify the A and B modes on the ΓX segment in spite of the photonic crystals being composed of anisotropic materials.

In Fig. 3(b), on the other hand, it should be noted that no mirror symmetry exists on the ΓX segment drawn by the arrow due to the distortion of the profile of the sixth band. Then, we cannot classify the A and B modes on the ΓX

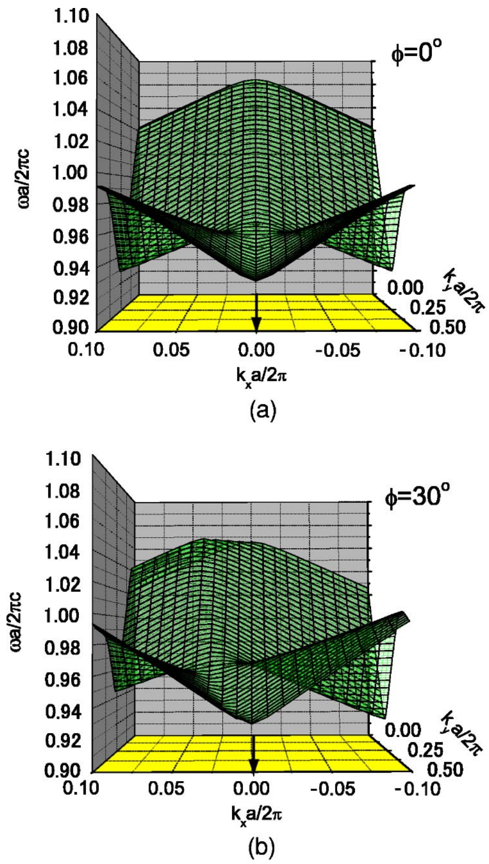

FIG. 3. Profiles of the sixth band when directors of liquid crystals are orientated at $\phi =$ (a) 0°, (b) 30°. The arrow indicates the Γ-X direction. The ordinary and extraordinary refractive indices are $n_{LC}^{o} = 1.522$ and $n_{LC}^{e} = 1.706$, respectively.

segment. That is, the uncoupled mode disappears in the photonic crystals.

In order to investigate the existence and disappearance of uncoupled modes, we calculate transmission spectra in two-dimensional photonic crystals when plane waves are incident, as shown in Fig. 1. We focus our attention on the shaded region in Fig. 2. Figures 4(a)–4(e) show transmittances in the Γ-X direction at $\phi = 0°$, 30°, 45°, 60°, and 90°, respectively. In Figs. 4(a) and 4(e), there exist the frequency regions in which transmittances are zero or very low. This is because mirror symmetry exists on the ΓX segment at $\phi = 0°$ and 90°. Electric field has an ordinary refractive index n_{LC}^{o} at $\phi = 90°$ although the electric field has an extraordinary refractive index n_{LC}^{e} at $\phi = 0°$. Properties of photonic crystals at $\phi = 90°$ become weaker than those of photonic crystals at $\phi = 0°$ because n_{LC}^{o} is lower than n_{LC}^{e}, and therefore, the transmittance is not zero in Fig. 4(e).

In Figs. 4(b)–4(d), on the other hand, no clear frequency regions of uncoupled modes exist in transmission spectra. This is because no mirror symmetry exists on the ΓX segment except at $\phi = 0°$ and 90° due to anisotropies of liquid crystals. However, it should be noted that there exist the frequency regions in which the transmittances are very low although the transmittances are very high in a certain frequency region. In Fig. 4(b), for example, the transmittance is

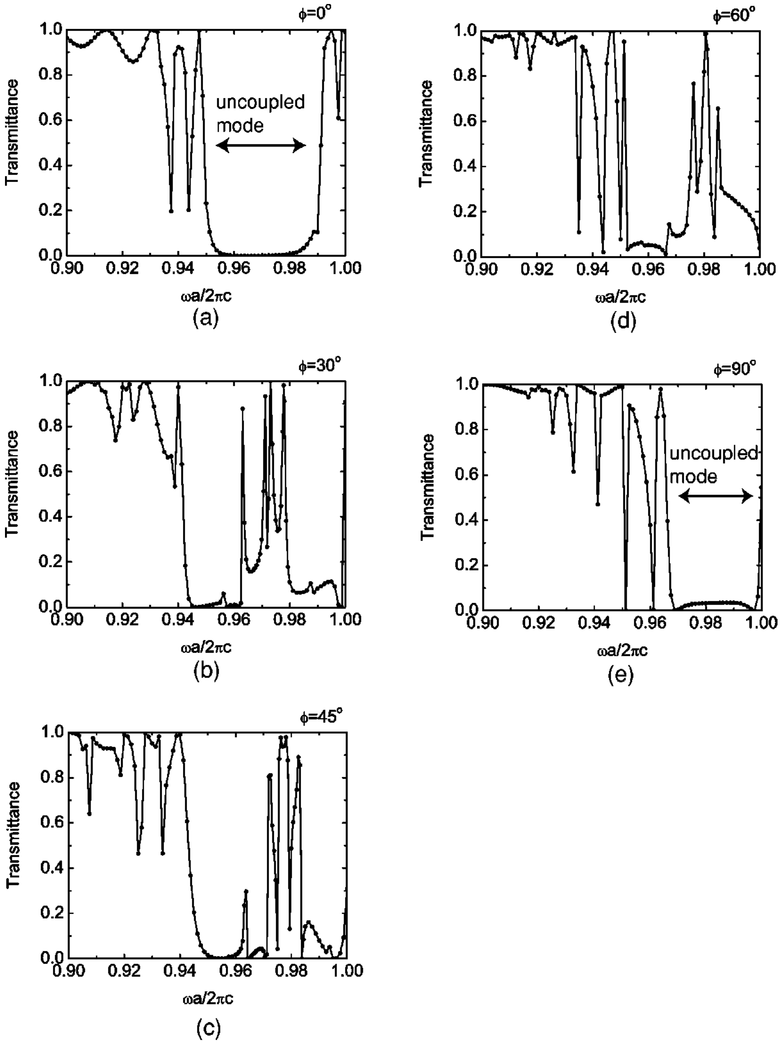

FIG. 4. Transmittance when directors of liquid crystals are orientated at (a) $\phi=0°$, (b) 30°, (c) 45°, (d) 60°, and (e) 90°, respectively.

very high in the frequency range of $(0.96–0.98)2\pi c/a$, which can be explained by the profiles of the sixth band in Fig. 3(b). As shown in Fig. 3(b), the profile of the sixth band is strongly twisted in the frequency range of $(0.96–0.98)2\pi c/a$ at $\phi=30°$. In such a frequency range, mirror symmetry greatly deteriorates, which causes the strong coupling of external plane waves. Except in such a frequency region, however, the profile of the sixth band is not strongly twisted. Therefore, there exist the frequency regions in which the transmittances are very high or very low.

In Fig. 5, moreover, we display the dependence of maximum transmittances resulting from the sixth band excited by external plane waves on ϕ ranging from 0° to 90°. As shown in Fig. 5, maximum transmittances are very high in the region of ϕ from about 10° to about 88°. The existence and disappearance of uncoupled modes correspond to opening and closing of photonic band gaps, respectively, in transmission spectra. In conventional photonic crystals, opening and closing of photonic band gaps need large changes of dielectric indices. By the use of anisotropies of liquid crystals, however, uncoupled modes become controllable. As mentioned above, the transmittances are zero or very low at $\phi=0°$ and 90° due to uncoupled modes. In transmission spectra, therefore, we could obtain large tunabilities by rang-

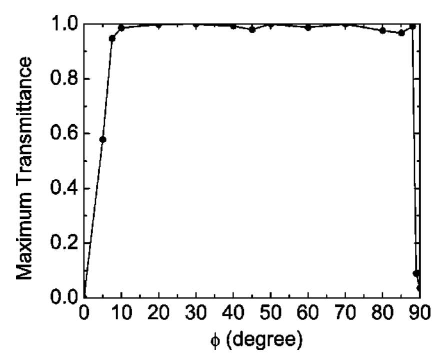

FIG. 5. Dependence of maximum transmittances resulting from the sixth band excited by external plane waves on ϕ ranging from 0° to 90°.

ing directors of liquid crystals from $\phi = 0°$ to $\phi = 10°$ or from $\phi = 90°$ to $\phi = 88°$.

IV. CONCLUSION

We demonstrated disappearances of uncoupled modes in two-dimensional photonic crystals due to anisotropies of liquid crystals theoretically. Mirror symmetry disappears in wave vectors by rotating directors of liquid crystals, which causes disappearances of uncoupled modes that cannot be excited by external plane waves. This property may provide large tunabilities in two-dimensional photonic crystals utilizing liquid crystals.

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